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Fictionalism and mathematical explanations

Ficcionalismo e explicações matemáticas

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ABSTRACT

In this paper, I place Mary Leng's version of mathematical instrumentalism within the context of the debate in mathematical realism/anti-realism as well as within the context of the platonism/nominalism debate. I maintain that although her position is able to show how the conjunction of Quinean naturalism and confirmational holism does not necessarily lead to the conclusion that mathematical objects must necessarily exist for they are indispensable in our best scientific theories; her usage of both theses still leads to platonism. Such is the case for her characterization of scientific theories as akin to a set-theory that accommodates fictitious objects and statements within it is untenable due to the dependence of fictions on a realist ontology.

Keywords: fictionalism, mathematical instrumentalism, indispensability argument, Mary Leng, platonism, nominalism.

RESUMO

Neste artigo, situo a versão de Mary Leng do instrumentalismo matemático no contexto do debate do realismo/antirrealismo matemático, bem como no contexto do debate do platonismo/nominalismo. Sustento que, embora sua posição seja capaz de mostrar como a conjunção do naturalismo quineano e do holismo confirmatório não leva necessariamente à conclusão de que os objetos matemáticos devem necessariamente existir, pois são indispensáveis em nossas melhores teorias científicas, seu uso de ambas as teses ainda leva ao platonismo. Esse é o caso de sua caracterização das teorias científicas, semelhante a uma teoria dos conjuntos que acomoda objetos e declarações fictícias dentro dela, sendo insustentável devido à dependência de ficções em relação a uma ontologia realista.

Palavras-chave: ficcionalismo, instrumentalismo matemático, argumento da indispensabilidade, Mary Leng, platonismo, nominalismo.

Introduction

Traditionally, philosophy of mathematics is characterized by the distinction between the foundational and anti-foundational positions in the field. Foundational positions which include formalism, intuitionism, and logicism maintain that (1) mathematics has a foundation and (2) mathematical development can be assessed using mathematical logic. Anti-foundational posi-

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tions, on the other hand, deny (1) and (2) as they maintain that mathematics and mathematical development can only be understood through a detailed analysis of mathematical practice. Contemporary arguments in philosophy of mathematics, however, deviate from this methodological and thematic distinction as they use logical tools in their assessment of how our conception of mathematical objects, statements, and knowledge is affected by mathematical practices. For this reason, there is a general agreement amongst the practitioners in contemporary philosophy of mathematics that the discipline has moved beyond the foundational/anti-foundational debates (Mancuso, 2008; Colyvan, 2012).

In line with this, philosophy of mathematics, as it is understood in this paper, includes arguments that emphasize the use of logical tools in the analysis of mathematical practice and methodology as well as their relation to the formation and justification of our most expedient scientific theories. Describing philosophy of mathematics this way allows a narrower yet more inclusive characterization of the field which continues to conform to the broader characterization of the discipline as the interpretation and illumination of the place of mathematics in the overall intellectual enterprise (Shapiro, 2005).

With this in mind, this paper delves into the contemporary debate in philosophy of mathematics between mathematical realism (MR) and anti-realism (MAR) about the ontology of mathematical objects. It addresses the tenability of Mary Leng's mathematical instrumentalism (MI) which posits that although adopting a naturalistic ontology shows the indispensable utility of mathematical objects in scientific theories, neither pure mathematics nor empirical science provide epistemic justification for the existence of mathematical objects. Hence, one should reject the existence of mathematical objects.

On the debate between mathematical realism/anti-realism

As of the moment, there are several aspects which characterize the debate between MR and MAR. An overview of the different positions in the debate shows the following. First, the distinction between MR and MAR is due to the differences of how they address the independence and existence dimensions of abstract objects which in turn define the independence, existence, and truth themes underlying the varieties of MR and MAR. MR and MAR, in this sense, can be understood in terms of their contrasting views about the role of human cognition in either the discovery or creation of mathematical objects and facts. Whereas the various forms of MR argue for the mind-independence of mathematical entities and/or facts, the different kinds of MAR argue otherwise. Michael Devitt (2008) refers to this as the independence and existence dimensions of the broader debate between all types of realisms and anti-realisms in philosophy. Although characterizing the debate through these metaphysical theses excludes both the epistemic and semantic theses, which some philosophers consider to be lacking in the standard characterization of the debate (*e.g.* Wright, 1992), there is a general consensus that the agreement or disagreement over these theses supplies the most useful characterization of the debate in philosophy. Such is the case since they are the overarching themes in the various forms of realisms and anti-realisms across the different domains in the discipline (Brock and Mares, 2007).

Second, contemporary debates between MR and MAR are divided into debates about ontological MR/MAR and truth-value MR/MAR. Shapiro (2005) maintains that the adaptation of the prevailing model-theoretic semantics in philosophy leads to either MR/MAR in ontology or MR/MAR in truth-value. The distinction between ontological and truth-value MR/MAR, in this sense, is a byproduct of the shift from the syntactic to the semantic view of theories.

Although both the syntactic and semantic view of theories focus on a theory's structure, they differ in their characterization of a theory's structure and the structure's relation to the world. The syntactic or sentential view of theories maintains that theories are deductively closed axiom systems which can be formalized using first-order logic. For example, in the syntactic view, a scientific theory T is the combination of its axioms AX and correspondence rules or coordinating definitions C. In comparison, the semantic or model-theoretic view of theories distinguishes between a theory's structure M and its linguistic formulation AX. M here is composed of a non-empty set U which is the universe or domain of the structure, an indexed set of operations O that is applicable to U, and a non-empty indexed set of relations R on U. In the model-theoretic view, M logically satisfies AX if M provides an interpretation where AX is true. M thereby functions as a truth-maker. In addition, M also represents a structure which is assumed to possess structural similarities to the world. In the semantic view, a model thereby realizes two functions. First, it interprets a theory's axioms and second, it represents the world thru a theory's model(s). In comparison to the syntactic view, the semantic view is supposedly free from the baggage of showing the correspondence of propositions, statements, or sentences to the world since it is mainly concerned with showing the structural similarities between a model and the world. Adopting the semantic view of theories, in this context, allows the distinction between ontological and truth-value MR/MAR since it allows one to determine the objectivity of a mathematical statement by positing that there is a model which gives it a true interpretation. In this scenario, the truth of AX is separated from empirical matters since the truth of AX is utterly dependent on the operations and relations allowed between the members of U. This paves the way for a view that maintains the objectivity of mathematical truth without necessarily presupposing the mind and language independent existence of the mathematical objects quantified in mathematical statements.

Within this context, those who adopt the difference between ontological and truth-value MR⁄MAR maintain that their dissimilarity lies in the former's focus on the quantified objects in a linguistic framework in comparison to the latter's focus on the truth-value of the well-formed meaningful sentences in a linguistic framework. Note, however, that even if the focus of both ontological and truth-value MR⁄MAR differs, a position's explanation for its commitments in the former directly affect its stance in the latter and vice versa. For example, he who adopts MR in truth-value may argue for the objectivity of mathematical statements due to the objectivity of logical consequence (Leng, 2010). He merely needs to emphasize the following. First, mathematical statements are products of logical reasoning and second, products of logical reasoning are objective due to the objectivity of logic. By highlighting these, one who adopts MR in truth-value argues for the objectivity of mathematical truths by maintaining that mathematical truths are logical consequences of the axioms of the mathematical theories from which they were derived. A problem, however, arises if one considers that logical consequence is explained through logical possibility. For instance, we maintain that given the axioms of a mathematical theory *T*, *Q* is a logical consequence of *P1, P2…Pn* if and only if it is not logically possible for *Q* to be false if *P1, P2…Pn* are true. Logical possibility, however, entails commitment to sets, a mathematical object. This is so because the best explanation for the logical possibility of Q states that Q is true since there is a set-theoretic model which interprets Q as true. From this argument, initially it seems that he who adopts MR in truth-value must also adopt MR in ontology.

I have noted this example to show my disagreement with Kreisel's dictum that "the point is not the existence of mathematical objects, but the objectivity of mathematical truth" (Kreisel, 1958, p. 138). In the example above, it is evident that shifting the focus on the objectivity of mathematical truth still leads to questions related to the existence of mathematical objects. In the example, it led to questions related to the existence of sets. It is for this reason that I have maintained that a position's explanations for its commitments in MR⁄MAR in truth-value directly affect its commitments in MR⁄MAR in ontology. A satisfactory account of mathematical knowledge and its applications thereby requires a coherent metaphysical account for both the truths and objects in mathematics.

From all these, it seems that, at first glance, one may adopt a seemingly intuitive position that MR⁄MAR in ontology leads to MR⁄MAR in truth-value and vice versa. However, such is not the case. Without delving into the merits of the following positions, a survey of the different positions in the debate shows that, as it is understood by most of the participants in the debate, ontological MR does not imply truth-value MR. In a similar manner, ontological MAR does not imply truth-value MAR. For instance, positions that deny the mind and/or language independence of mathematical objects and hence the literal interpretation of mathematical statements either maintain that mathematical statements are true (*e.g.* Hellman's modal structuralism), false (*e.g.* Field's fictionalism), or do not have a fixed truth-value (*e.g.* Dummett's intuitionism). Likewise, positions that affirm the existence of mathematical objects and hence the literal interpretation of mathematical statements either maintain that mathematical statements are true (*e.g.* Godel's logicism) or false (*e.g.* Tennant's intuitionism).

Let us now move on to the next two characteristics of the debate. Third, contemporary debates in ontological MR⁄MAR begin by either justifying or debunking the adoption of a Quinean naturalist methodology in determining the existence of mathematical objects. Lastly, MR/MAR in ontology address the issues surrounding the existence of mathematical objects by initially providing their views about epistemic justification in science. I will expound on both of these points in the development of the paper's argument. For now, suffice it to state that regardless of the availability of other methodologies to both sides, it is generally considered that Quinean naturalism best demonstrates the relationship between the issues related to the ontological existence of mathematical statements, the truth-value of mathematical statements, and the applicability of mathematics outside the discipline.

Given the varieties of both MR and MAR, this paper focuses on the nominalist and platonist positions in the debate in MR⁄MAR in ontology. The most prominent positions on the side of MR include the different versions of platonism, physicalism, and psychologism. The most well-known positions on the side of MAR, on the other hand, include the various forms of formalism, implicationism, and nominalism. This paper focuses on the nominalist and platonist arguments, however, since they provide the most viable versions of the third and fourth characteristics of the debate which I mentioned earlier. Recall that the third characteristic of the debate highlights how arguments between MR/MAR in ontology either justify or debunk the adoption of a Quinean naturalist methodology as they either prove or disprove the existence of mathematical entities. Remember as well that the fourth characteristic of the debate stated above highlights how the justification or refutation of the use of Quinean naturalism provided by those who adopt either MR⁄MAR in ontology leads those who participate in the debate to initially supply their views about epistemic justification in science. Both of these points define one of the primary interests of this paper, which is the implications of the methodology one adopts in one's argument for either the existence or non-existence of mathematical entities.

An overview of the arguments given by nominalists and platonists show that the demarcation between both groups lies in the former's objections against the latter's justification for the existence of abstract objects. Platonists argue for the existence of *abstracta* whereas nominalists remain skeptical, if not adamant, about the existence of non-spatial, non-temporal, causally impassive and inactive objects (Burgess and Rosen, 1997). A loose formulation of their arguments will show that platonists adopt a Quinean naturalist methodology in order to use the indispensability argument (IA) to determine the existence of mathematical objects. In fact, platonists'arguments for the third and fourth characteristics of the debate mentioned earlier are dependent on their development of their IAs. This has led to a variety of IAs.

What is important to note in relation to the variety of IAs is how they emphasize the role of mathematical explanations in proving the existence of mathematical objects. The IA, in its traditional formulation, is characterized by its emphasis on the quantification of mathematical objects in empirical theories which is a result of its adherence to holism. By adopting a holistic view of theories, it presupposes the existence of a linguistic framework that determines the inferential connections between the statements within it. In general, holism argues that an utterance can only be meaningful within the context of the linguistic framework where it derives its content. As a result, a broad interpretation of holism shows that a theory's confirmation leads to the equal confirmation of its non-eliminable claims, including mathematical statements. Most of the versions of the traditional formulation of the IA follow this route, as can be seen in Michael Resnik's presentation of the traditional IA below:

> *(1) Mathematics is an indispensable part of our best scientific theories.*
> *(2) Mathematics shares whatever confirmation accrues to the theories using it (Quinean holism).*
> *(3) So mathematics shares the confirmation accruing to our best scientific theories.*
> *(4) We are committed to the truth of our best scientific theories (naturalism).*
> *(5) So we are also committed to the truth of the mathematics they contain (Resnik, 2003, p. 232).*

In general, the positions that generate problems with this view can be determined in line with the platonist-nominalist distinction in ontological realism. Platonists, for instance, in favoring IA, may still take issue with (2) and (3). Nominalists, on the other hand, may pose problems for (1) to (5) with the exception of (4).

In most cases, concerns against the traditional formulation of IA are traced to its holistic framework. Recent configurations of IA, like Resnik's, thereby exclude (2) and (3) in the formulation of the argument. Colyvan's IA, for instance, highlights the role of indispensability as it posits:

> *(1) We ought to have ontological commitment to all and only those entities that are indispensable to our best scientific theories.*
> *(2) Mathematical entities are indispensable to our best scientific theories.*
> *(3) We ought to have ontological commitment to mathematical entities (Colyvan, 2001, p. 11).*

Colyvan characterizes indispensability via the characteristics of dispensability, wherein he considers a theory dispensable if and only if:

> *(1) There exists a modification of the theory in question resulting in a second theory with exactly the same observational consequences as the first, in which the entity in question is neither mentioned nor predicted.*
> *(2) The second theory must be preferable to the first (Colyvan, 2001, p. 11).*

He thereby argues for the indispensability of mathematics in scientific theories by appealing to its preferability over other theories on the grounds of its simplicity, our familiarity with its principles, the wider scope of its application, its fecundity, and its testability. The existence of mathematical objects, in this sense, is determined through IBE.

Another recent argument which uses IBE is Alan Baker's enhanced indispensability argument (EIA). EIA can be seen as the byproduct of the agreement between platonists and nominalists that the indispensability of mathematics in scientific theories ought to account for the explanatory role of mathematics in science (Baker, 2009; Colyvan, 2002). It assumes:

> *(1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.*
> *(2) Mathematical objects play an indispensable explanatory role in science.*
> *(3) Hence, we ought rationally to believe the existence of mathematical objects (Baker, 2009, p. 613).*

In comparison to Colyvan's IA, EIA utilizes IBE while emphasizing the inseparability of mathematics from empirical science by arguing for its representational and explanatory role in successful scientific theories.

To support their assumptions, both Colyvan and Baker formulated case studies that apply their IAs to scientific theories. Baker argues for the indispensability of a number theoretic theorem in the explanation of a purely physical phenomenon, the life-cycle of the North-American cicadas. In conjunction with the premises of EIA, Baker includes the following in his analysis of the North American cicada's life-span:

> *(1) Having a life-cycle period that minimizes intersection with other (nearby/lower) periods is evolutionary advantageous. (biological law)*
> *(2) Prime periods minimize intersection (compared to non-prime periods). (number theoretic theorem)*
> *(3) Hence organisms with periodic life cycles are likely to evolve periods that are prime. ('mixed' biological/mathematical law)*
> *(4) Cicadas in ecosystem-type E are limited by biological constraints to periods from 14 to 18 years. (ecological constraint)*

> *(5) Hence cicadas in ecosystem-type E are likely to evolve 17-year periods (Baker, 2009, p. 614).*

Baker maintains that (2) is an essential component in the explanation of the cicada's life-span since it provides explanatory power by showing how primeness is a necessary component in explaining the evolution of the *Magicacicadas*. In addition, he considers it a genuine mathematical explanation since it is an application of a number theoretic theorem to a physical phenomenon which needs explanation. In relation to the existence of mathematical entities, Baker maintains that since (2) provides genuine explanatory power to a scientific theory which shows its indispensability in the theory, it also proves, using IBE, the existence of prime numbers.

In comparison to Baker's case study, Colyvan and Lyon (2008) inquire on (1) the role of Hale's honeycomb theorem in explaining the hexagonal division of beehives and (2) the role of the Hamiltonian formulation in explaining the Henon-Heiles system. In (1), along with Baker, they also distinguish between the scientific component, in this case the evolutionary component, and the mathematical component behind the phenomena as they claim:

> *(T)he biological part of the explanation is that those bees which minimise the amount of wax they use to build their combs tend to be selected over bees that waste energy by building with excessive amounts of wax. The mathematical part of the explanation then comes from what is known as the honeycomb conjecture: a hexagonal grid represents the best way to divide a surface into regions of equal area with the least total perimeter (Colyvan and Lyon, 2008, p. 230).*

Given Colyvan's characterization of dispensable theories, it is assumed that by IBE the explanation for the hexagonal division of beehives supplied by the combination of the explanations from Darwin's theory of evolution and Hale's honeycomb theorem proves the existence of mathematical objects. Such is the case since the honeycomb theorem supplies the best mathematical explanation for the phenomena. In the case of (2), on the other hand, Colyvan and Lyon (2008) demonstrate that a reformulation of the theory leads to the loss of its explanatory power since it excludes the explanation of phase-space and the Poincare map.

Within this context, in line with the third and fourth characteristics of the MR/MAR debate mentioned earlier, several characteristics of the contemporary formulations of IA can be derived from Colyvan's (2001; 2008) and Baker's (2009) versions, these being: (1) their separation of IA from holism, (2) their use of IBE, and (3) their emphasis on the representational and explanatory role of mathematics in successful empirical theories. From these characteristics, either (2) or (3) or both are the targets of the counter-arguments against these positions. Counter-arguments against both positions that question (2) reject the applicability of IBE to mathematical posits. Those who question (3), on the other hand, desire a clear demarcation of the representational role from the explanatory role of mathematics in successful scientific theories. Finally, arguments against both (2) and (3) present either a combination of the reasons for the rejection of (2) or (3) or target their reliance on scientific realism.

Other criticisms of these contemporary versions of IA point out that the scientific explanation of a physical phenomenon merely provides its mathematical representation. It is indeed the case that in the formalization of a scientific theory, one adopts a particular mathematical model. Hence, criticisms that take this form argue that contemporary formulations of IA need to explain the relationship between mathematical models and scientific reality before indicating that there is the need to discuss the role of mathematical explanations in successful empirical theories. Shapiro states this succinctly as he claims:

> *Clearly, a mathematical structure, description, model, or theory cannot serve as an explanation of a non-mathematical event without some account of the relationship between mathematics per se and scientific reality. Lacking such an account, how can mathematical/scientific explanations succeed in removing any obscurity – especially if new, more troubling obscurities are introduced? (Shapiro, 2005, p. 217).*

An example of a perspective which addresses the problems of establishing the relationship of the theory and its model can be seen in Christopher Pincock's mapping account of mathematical application which maintains that for a mathematical model to represent a scientific theory it must indicate "(i) what purely mathematical entity or structure is in question, (ii) how the parts of the mathematics are to be physically interpreted and (iii) what sort of structural relation must obtain between the interpreted mathematical structure and the target system for the claim or model to be accurate" (Pincock, 2010, p. 3). In Pincock's (2010) view, the main goal of applied mathematics is representational. Criticisms of this view emphasize that it delimits the application of mathematics to its capacity to provide mathematical models of successful scientific theories. The role of applied mathematics, however, is not limited to this as it also aids in the formulation of new predictions as well as in the explanation of phenomena.

Regardless of the variety of IAs formulated by platonists nowadays and the criticisms forwarded against them, these IAs remain similar as they use the premises of scientific realism to argue for (1) the mind and language independence as well as (2) the literal interpretation of the parts of mathematical discourse used in scientific discourse. It is for this reason that platonists maintain that (1) the existence of mathematical objects can be derived from the literal truth of scientific theories and (2) there are good reasons to assume the literal truth of most scientific theories. Nominalists, on the other hand, in their adoption of a Quinean naturalist methodology also utilize SR. However, in contrast to platonists, they argue for (1) the mind and/or language dependence and/or (2) the non-literal interpretation of the parts of mathematical discourse used in scientific discourse. It is for this reason that nominalists maintain that (1) mathematical objects are incorporated in our best scientific theories only for their instrumental value and (2) there are good reasons to assume the empirical adequacy of scientific theories as opposed to their literal truth.

An analysis of Mary Leng's mathematical fictionalism

In the case of Leng's MI, she adopts a nominalist position as she argues for the non-existence of mathematical objects by arguing for the coherence of MI over scientific realism (SR) and constructive empiricism (CE). Her MI is primarily a refutation of MR which she equates with both platonism and the combination of SR and IA. She maintains:

> *(S)ince scientific realism so understood by itself implies that we ought to believe in all the objects indispensably posited to exist by the statements we use to express our best scientific theories, scientific realism combined with the indispensability of mathematics automatically implies mathematical realism […] I will use the label mathematical realism interchangeably with the label "platonism," where platonism is the view that we ought to believe in mathematical objects understood as abstracta […] If we take (an) entirely negative characterization of abstracta, then any version of mathematical realism that denied the abstractness of mathematical objects would have to argue that mathematical objects are spatially or temporally located, or that they are causally efficacious […] I take it to be safe to assume that belief in mathematical objects amounts to belief in abstracta negatively characterized (Leng, 2010, p. 9-10).*

Initially, what is important to recognize in Leng's characterization of MR is how she highlights the connection between SR and MR. By showing that SR combined with IA leads to MR, she shows that questions regarding the existence of mathematical objects are not only of interest in philosophy of mathematics but also in philosophy of science. This is so since if one maintains that our most expedient scientific theories are true or approximately true, then one is committed not only to the truth of the mathematical statements in those theories but also to the existence of the mathematical objects posited in those theories. It is for this reason that Leng argues for the coherence of MI over SR and CE. Another aspect of Leng's MI which is important to consider is how it responds to one of the concerns mentioned in the previous section, that being the need to supply an account of how mathematics is able to represent scientific reality.

Within this context, Leng uses Quine-Putnam's indispensability argument (QPI) as the focal point of her refutation of MR. She presents QPI in the following form:

> *P1 (Naturalism): We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe.*
> *P2 (Confirmational Holism): The confirmation our theories receive extends to all their statements equally.*
> *P3 (Indispensability): Statements whose truth would require the existence of mathematical objects are indispensable in formulating our best confirmed scientific theories.*
> *C (Mathematical Realism): We ought to believe that there are mathematical objects (Leng, 2010, p. 7).*

She also provides the main assumptions of Bas van Fraassen's CE, these being:

(1) One ought to adopt the mathematically stated theoretical world-pictures provided by science.
(2) Scientific theories are mathematically stated theoretical world-pictures since they "represent empirical phenomena as embeddable in certain abstract (mathematical) structures" wherein these abstract mathematical structures are "describable only up to structural isomorphism" (Van Fraassen, 2010, p. 238).
(3) Scientific theories are not necessarily true, they are only empirically adequate wherein a scientific theory is true if it "has a model which is a faithful replica, in all detail, of our world" (Van Fraassen, 1980, p. 68-69) and a scientific theory is empirically adequate if it has some "surface (data) models of phenomena (that) fit properly with or into the (theory's) theoretical models" (Van Fraassen, 2010, p. 305).
(4) Within scientific theories, a hypothesis has an objective truth-value not because it supplies a literal description of the world but because it is within a theory which fits an adequate representation of a phenomenon.
(5) Within scientific theories, only hypotheses about observable objects and phenomena, objects and phenomena which can be observed in the circumstances contextually implied within scientific theories, have objective truth-values.

In relation to CE's position regarding the existence of mathematical objects, it subscribes to a nominalist position as it remains agnostic about the existence of mathematical objects. In the case of QPI, Leng maintains P1 and P3 while denying P2. In the case of CE, although Leng concurs with (1) and (2), she denies (3), (4), and (5).

Before proceeding to the basic assumptions of Leng's MI to initiate the comparison of MI with QPI and CE, it is important to note that amongst the assumptions of CE mentioned above, I derived (2), (3), and (4) from Van Fraassen's SE. In this regard, within her works, Leng only explicitly accepts (1). In the same manner, she only explicitly denies (3) and (5) based on Van Fraassen's initial description of empirical adequacy in his CE. As one will note, however, based on the assumptions of Leng's MI presented below, she can be understood as accepting (2) and denying (3) and (4) of CE as well.

Within this context, an overview of her argument for MI, given her acceptance of some of the assumptions of QPI and CE, allows me to formulate her overall argument in the following way:

(1) We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe.

(2) A reflective understanding of science and scientific practice requires us to adopt the norms of science, which in the case of ontological scruples, is the norm of parsimony that delimits the entities assumed to exist within theories to those which must exist by necessity.

(3) The confirmation our scientific theories receive shows the instrumental value of adopting a mathematical model for our scientific theories based on a prop-oriented theoretical framework that distinguishes, based on its principles of generation, between statements adopted for their utility as opposed to statements adopted for their literal truth.

(4) The confirmation our mathematically stated hypotheses receive only confirms their status as theoretical fictions.

(5) Mathematical statements whose fictional truth would require the existence of mathematical objects are indispensable in formulating our best confirmed scientific theories.

C: A reflective understanding of science provides reasonable grounds to reject the existence of mathematical objects.

It is important to note that I focused on her agreement with some of the assumptions of QPI and CE in presenting her MI since they provide what Leng considers to be the most viable arguments that lead to either MR or MAR. In the case of QPI, it depicts the combination of naturalism and IA, the trademark of MR. In the case of CE, Leng shows that there is no strict disjunction between SR and CE. She argues that to deny CE does not entail adherence to SR in the same way that to adopt naturalism does not entail subscription to SR. Leng's position, in this sense, can be understood, to a limited extent, as a midway position between MR and MAR. She considers her view to be subscribed under a limited-scientific anti-realism as she argues for the existence of some unobservable physical objects due to her acceptance of the application of inference to the best explanation (IBE) for both observable and unobservable physical objects but not for abstract objects such as mathematical entities.

Given these assumptions, an initial analysis of Leng's argument shows that her MI espouses the following:

Ontological Assumption (OA): There exists a mind-independent world which is composed of non-mathematical objects, these being observable and unobservable physical objects.

Semantic Assumption (SA): Only the nominalistic content, the non-mathematical content, of a theory possesses an objective truth-value.

Epistemic Assumption (EA): Scientific theories with great predictive and explanatory power provide nominalistically adequate representations of reality wherein a theory T provides a nominalistically adequate representation of reality if and only if its nominalistic content corresponds with the physical world.

Along with these, Leng's (2005; 2010) MI allows us to maintain that her views reflect the following attitudes towards mathematics:

Anti-hermeneuticism: Mathematical theories are not truth-normed for these theories merely provide the logical consequences of adopting our characterizations of mathematical concepts regardless of mathematicians' different attitudes towards mathematical practice.

Anti-revisionism: Mathematical practice should not be revised regardless of the results of philosophical criticisms.

Doublethink: It is appropriate to suspend belief in the existence of mathematical objects in both mathematical and scientific practice as long as one recognizes that philosophy provides justified reasons to reject the existence of mathematical objects.

Given these assumptions, the appeal of addressing Leng's MI lies in its presentation of a new perspective not only for understanding the relationship of both pure and applied mathematics to empirical science but also for addressing the issues surrounding the debate between ontological MR and MAR. In addition, her *Mathematics and Reality* also gives the most comprehensive discussion of MI in the existing literature (Leng, 2010). Leng fills a lacuna in the existing literature about ontological realism as she supplies a fictionalist's stance for both pure mathematics and science. In the process, she shows that one can adopt a naturalist methodology and argue for the indispensability of mathematics in scientific theories even if one subscribes to nominalism. As a result, she supplies those who adhere to MR, merely due to their disagreement with CE, an alternative stance for developing their arguments for the non-existence of mathematical objects.

In spite of these, Leng's MI seems incoherent for the following reasons. First, it is necessary to distinguish between a naturalist methodology and a fictionalist methodology in so far as a naturalistic methodology cannot be separated from a view which presumes that the goal of scientific inquiry is truth, and not merely empirical adequacy. Second, it is necessary to distinguish between the representational as opposed to the explanatory role of mathematics in our most expedient scientific theories in order to address MI's presumption that there is a clear demarcation that can be made between the fictional as opposed to the literal content of these theories. Finally, it is also necessary to clarify the background assumptions of fictions understood as metaphors in order to clarify MI's presumption that fictions do not require a realist ontology.

The need to distinguish between a naturalist as opposed to a fictionalist methodology seems to be implied within Leng's development of MI itself as she maintains that her MI is "profoundly un-Quinean" as it opts for the revival of the Carnapian sense of the analytic-synthetic distinction (Leng, 2010, p. 260). Though arguably Quinean naturalism still retains some aspects of Carnap's conventionalism without the two dogmas of empiricism, it is still important to flesh out how her MI's interpretation of naturalism requires a distinction between a naturalist as opposed to a fictionalist methodology in order to reconcile what seems to be an incoherence in terms of her MI's adoption of naturalism and her MI's views about scientific inquiry.

Before elaborating on the second and third reasons for the incoherence of Leng's MI mentioned above, it is important to trace how they are made possible by her adoption of Kendall Walton's claim that fictional discourse is comparable to a game of pretence or make-believe. In line with her presentation of Walton's theory of make-believe, Leng (2010) forwards the following claims:

(1) A discourse is fictional so long as there is a prescription to imagine that it is true regardless of whether it is true or false.
(2) Prescriptions of what can be imagined within a fictional discourse are determined by the rules of generation of that discourse which in turn determine the claims that are allowed and disallowed within that fictional discourse.
(3) A fictional discourse can be prop oriented wherein a game of make-believe or pretence is prop oriented if the primary interest of participating in the discourse is to describe and supply information about objects independent of the discourse.

Applying these assumptions to our most expedient scientific theories, Leng maintains that scientific discourse is a prop-oriented fictional discourse following the rules of generation of the game of set theory with non-mathematical urelements. She states:

> *We need […] to tell some story about how mathematical objects relate to non-mathematical objects, so as to allow us to view our mathematically stated scientific laws as hypotheses concerning what is fictional in this story […] We have such a story, in set theory with non-mathematical objects as urelements […] (In this story), facts about non-mathematical objects can generate the fictionality of utterances in the context of set theory with urelements […] (H)ow things are with the non-mathematical props will make fictional some utterances in the context of this make-believe. And similarly, by hypothesizing that a given utterance is fictional in this make-believe, we can indirectly represent a hypothesis concerning how things are with the non-mathematical props (Leng, 2010, p. 176-178).*

In this case, adopting Walton's notion of prop-oriented make-believe allows Leng's MI to do the following. First, distinguish between the fictional as opposed to the literal content of a theory. Second, distinguish between the instrumental as opposed to the descriptive content of a theory. Finally, argue for the nominalistic adequacy of our most expedient scientific theories.

Within Leng's MI, instead of claiming the truth of our most expedient scientific theories, one is only warranted in maintaining that an expedient scientific theory is nominalistically adequate. Such is the case since it is only the non-mathematical content of the scientific theory which represents the conditions in the world. In other words, it is only the non-mathematical content of our most expedient scientific theories which can be understood as literally true since its mathematical content is merely true within the fiction of set theory with non-mathematical urelements. Adopting the game of set theory with non-mathematical urelements thereby allows Leng's MI to further distinguish between the descriptive as opposed to the instrumental parts of a scientific theory. It is only the non-mathematical content of the scientific theory which has a descriptive function, whereas the mathematical content of the scientific theory only has an instrumental function. It is instrumental in so far as it allows one to indirectly determine, by virtue of how one characterizes the mathematical content of a theory, which objects and statements within the scientific theory count as mathematical and non-mathematical.

Leng's MI, however, falls short in terms of its method for distinguishing between the fictional as opposed to the literal content of a scientific theory in so far as she presumes that there is a fixed interpretation that will distinguish between the mathematical content and the non-mathematical content within a given scientific theory. An observation of scientific practice, however, shows us that such is not the case (*e.g.* kinetic theory of heat). In the initial stages of a scientific theory's formulation, there is no fixed demarcation between its mathematical and non-mathematical content. This is so not because one cannot access the non-mathematical content of a theory, but because what we classify as the non-mathematical content of a scientific theory is dependent on our current understanding of natural phenomena.

As a result, it seems that setting the dividing line between what is literal as opposed to fictional in a scientific theory can only be done by MI retrospectively. That is, when a scientific theory has already successfully predicted and explained a particular phenomenon. To solve this, it is important to clarify how Leng's MI distinguishes between the representational as opposed to the explanatory role of mathematics in scientific theories. Such is the case since, at best, her claim that scientific theories ought to be understood within the game of set theory with urelements only shows how a mathematical game may represent or describe the distinction between the mathematical and non-mathematical objects within a scientific theory. It seems that it cannot supply an adequate reason for how the inclusion of a mathematical explanation within a scientific theory supplies that theory with greater predictive and explanatory power.

This brings to light another problem with Leng's MI which is dependent on her assumption that there is an equivalence between fiction and metaphor. Leng follows Yablo as she adopts Yablo's characterization of a metaphor as "an utterance that represents its objects as being *like so*: the way that they *need* to be to make the utterance pretence-worthy in a game that it itself suggests" (Yablo, 1998, p. 170). Her adoption of the equivalence between fiction and metaphor is also apparent as she claims:

> *On the account we have been considering, by uttering a sentence S that appears to concern the relation between real, concrete, objects and the objects of some ideal model, a theorist may succeed in representing some metaphorical content of S to be true. And if so, the value of uttering the sentence S might be indirectly to assert not that S is true, but rather, to assert S's metaphorical content, that is, that the real worldly 'props' in the game are such as to make the sentence S fictional (Leng, 2010, p. 170-171).*

In this context, since her MI maintains that mathematical statements are akin to metaphors, her MI is able to maintain the distinctions between instrumental as opposed to descriptive and fictional as opposed to literal statements mentioned above. Restating scientific stories in the game of set theory with urelements thereby allows her MI to highlight facts that hold outside the game. In other words, it allows her MI to maintain that pretending as if mathematical objects share the same ontological status of non-mathematical objects allows one to elicit facts about the non-mathematical objects included in the game. In this case, mathematical objects and statements are indispensable to the formulation of our most expedient scientific theories since they allow us to infer facts about non-mathematical objects. To this extent, her association of fictions with metaphors serves as a backdrop for her MI's indispensability claim.

However, as I see it, anchoring her indispensability claim on the comparison of fictions with metaphors leads to a problem mainly because fictions understood as metaphors require a realist background ontology. To begin with, her MI's presupposition that there is a well-drawn distinction between the fictional as opposed to the literal claims within a scientific theory already presupposes that even if scientific theories are understood within a game which already hypothesizes, as a result of its rules of generation, the fictional claims within the game, it is still the scientific theory's prediction and explanation of phenomena which allows one to infer that a set of claims are fictional as opposed to another set of claims that supply literally true descriptions of scientific reality. The game, however, proceeds as scientists pretend that mathematical objects share the same ontological status of the non-mathematical objects within the game. A realist background ontology, in this case, is required in comparing fictions with metaphors since one must already presuppose that the properties attributed to non-mathematical objects can also be attributed to mathematical objects. Such a presumption, however, is problematic since, to begin with, in the state wherein a scientific theory has yet to successfully predict and explain a phenomenon, one is unclear about the properties that both the mathematical and non-mathematical objects may share. One cannot appeal to the usual characterization of abstracta here since Leng maintains that her MI is also committed to the existence of unobservable physical objects.

Conclusion

In this paper, I provided (1) an overview of the debate in MR/MAR while (2) at the same time showing the value of Leng's MI. However, (3) I have also pointed out that regardless of whether Leng shows that mathematical statements in our best scientific theories can be treated as akin to fictions, the primary problem with her account is that fictions require a realist ontology. Hence, (4) even if she characterizes the game of make-believe as occurring within a particular set theory with urelements, it would also require that the components of this set theory and its primary elements should be understood in the context of a realist ontology. This leads her nominalist view to be privy again to platonist criticisms. Regardless, (5) Leng's MI is still valuable since it seems to steer us to a variant of a constructive empiricist view contrary to her claim. That is, (6) if set theory is understood as a game of make-believe in order to explain away the discrepancy in the existence of idealizations in our best scientific theories, we may claim that the game of set theory actually provides us with a framework that has similarities to the actual external world that our scientific theories aim to represent. (7) This opens up further discussion on how the debate between MR/MAR can be connected to the ongoing debates in philosophy of mind regarding how we should characterize consciousness and to what extent consciousness allows us to access the external world.

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